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Global analyses of evolutionary dynamics and exhaustive search for social norms that maintain cooperation by reputation

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Abstract

Reputation formation is a key to understanding indirect reciprocity. In particular, the way to assign reputation to each individual, namely a norm that describes who is good and who is bad, greatly affects the possibility of sustained cooperation in the population. Previously, we have exhaustively studied reputation dynamics that are able to maintain a high level of cooperation at the ESS. However, this analysis examined the stability of monomorphic population and did not investigate polymorphic population where several strategies coexist. Here, we study the evolutionary dynamics of multiple behavioral strategies by replicator dynamics. We exhaustively study all 16 possible norms under which the reputation of a player in the next round is determined by the action of the self and the reputation of the opponent. For each norm, we explore evolutionary dynamics of three strategies: unconditional cooperators, unconditional defectors, and conditional cooperators. We find that only three norms, simple-standing, Kandori, and shunning, can make conditional cooperation evolutionarily stable, hence, realize sustained cooperation. The other 13 norms, including scoring, ultimately lead to the invasion by defectors. Also, we study the model in which private reputation errors exist to a small extent. In this case, we find the stable coexistence of unconditional and conditional cooperators under the three norms.

Keywords: Indirect reciprocity; Reputation; Norms; Discriminator; Exhaustive search; Replicator dynamics

1. Introduction

Richard Alexander (1987) said that indirect reciprocity is "a consequence of direct reciprocity occurring in the presence of interested audiences". The audiences repeatedly evaluate members in a society and judge who deserves help. Those who gain a good reputation receive donation from others while those who gain a bad reputation miss help. Cooperative act is passed from person to person via reputation. Hence, having good reputation or status is of great importance in indirect reciprocation (Fehr, 2004). Recently, much empirical work has been done to reveal the nature of indirect reciprocity and reputation formation in humans (e.g. Bolton et al., 2005; Milinski et al., 2001, 2006; Wedekind and Milinski, 2000; Wedekind and Braithwaite, 2002). For a recent review on indirect reciprocity, we refer to Nowak and Sigmund (2005).

In theory, Nowak and Sigmund (1998a, b) investigated how indirect reciprocity works among individuals. Nowak and Sigmund (1998b) introduced binary reputation, either good or bad, to represent the social status of players. Individuals repeatedly play a Prisoner's Dilemma game with others, each time recruiting a different opponent from the society. There are two strategic choices in this game, cooperation or defection. Those who cooperate pay cost c for their opponent to receive benefit b. Those who choose defection pay nothing. Players do not interact with the same person more than once. According to the result of the game, observers assign a new reputation to players. The way to attach reputation is as follows. Those who cooperated in the previous interaction receive a good reputation. In contrast, those who refused to help others in the previous round gain a bad reputation. This rule of assigning reputation, or namely the "norm", is called

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"scoring" (Brandt and Sigmund, 2004; Ohtsuki and Iwasa, 2004). Nowak and Sigmund (1998b) showed that under scoring conditional cooperators who help only good individuals are resistant to defectors, because they selectively give help only to cooperative members. Reputation works as media for community enforcement (Kandori, 1992).

The simplest as it is, scoring has a critical shortcoming in it. It cannot distinguish sanction from selfish defection. Scoring assigns a bad reputation to conditional cooperators who refused to help a bad person as punishment. Therefore, no conditional cooperators are motivated to give cheaters a penalty, which obviously leads to a triumph of defectors. Previous theoretical works confirmed that scoring is not able to sustain cooperation under errors (Panchanathan and Boyd, 2003, Ohtsuki, 2004, Ohtsuki and Iwasa, 2004) without an additional mechanism, such as growing social networks (Brandt and Sigmund, 2005).

Following this result, Ohtsuki and Iwasa (2004, 2006) searched for combinations of a norm and a behavioral strategy that can maintain cooperation, among a huge number of possibilities. According to their formulation, a norm judges whether an observed action is good or bad taking the following three components into account: (i) the action of the focal player (cooperation or defection), (ii) the reputation of the opponent (good or bad), and (iii) the reputation of the focal player (good or bad). This type of norm is called third-order assessment (Brandt and Sigmund, 2005). Also, a player's behavioral strategy that prescribes the action (to cooperate or to defect) toward an opponent is conditional on (a) the reputation of the opponent (good or bad), and (b) the reputation of the focal player (good or bad). In this framework, Ohtsuki and Iwasa (2004) asked under which norm which behavioral strategy becomes an evolutionarily stable strategy (ESS) that realizes cooperation at a high level even under a small amount of errors. As a result, they found eight combinations of a norm and a behavioral strategy, called the "leading eight", which were characterized in the subsequent paper (Ohtsuki and Iwasa, 2006).

While exhaustive ESS analyses have been completed by Ohtsuki and Iwasa (2004, 2006), their analysis is restricted to the invasibility of the equilibrium dominated by a single strategy. No works have conducted a global analysis of evolutionary dynamics of strategies over all possible norms. One of the aims of the present paper is to obtain a complete classification of evolutionary dynamics over all possible norms. Here, we focus on the norms that are based on second-order assessment instead of third-order assessment (Brandt and Sigmund, 2005). That is, a norm in this category specifies whether a player is good or bad based on (i) the action of the focal player (cooperation or defection), and (ii) the reputation of the opponent (good or bad), but without using the previous reputation of the focal player. Similarly, we consider behavioral strategies that are conditional only on the reputation of the opponent (good or bad), but not on the reputation of the self. By such a simplification, we can reduce the total number of norms from 256 to 16 and the total number of behavioral strategies from 16 to 4, making the exhaustive examination of global behavior feasible. Out of all 16 conceivable norms we find that three norms, called "simple-standing", "Kandori", and "shunning", realize sustained cooperation.

We also study the effect of private reputation errors in evaluating others in indirect reciprocation. If players individually and independently commit errors they result in having different opinions on the same person. Hence, reputation is not public information but can be a private opinion (that is, we must consider not only *who is good* but also *who thinks who is good*). Here, we study the effect of private reputation errors under a very simple assumption. The reputation of all the members in the population is determined publicly, but there is a small chance that each player makes a mistake in interpreting the reputation of others. We show that the existence of such private reputation errors induces the stable coexistence of unconditional and conditional cooperators under the three norms mentioned above.

2. Model

2.1. Prisoner's dilemma game

Consider an infinitely large population. For each integer round $t = 1, 2, \ldots,$ each player randomly finds an opponent and engages in a one-shot Prisoner's Dilemma game. There are two behavioral choices, either to give help (= C; cooperation) or to refuse help (= D; defection). Cooperation costs c to the donor and yields the benefit b to the recipient. In contrast, defection yields nothing to either. Two players in a pair decide their actions simultaneously and gain payoffs. After the game they leave the pair, and each of them seeks an opponent in the next round. The number of rounds played in a generation by each player follows a geometric distribution. Parameter ω represents the probability that the next round exists ($0 \leqslant \omega < 1$). Hence, the mean payoff is calculated as the summed payoffs in which future gains are discounted by the factor ω per round.

2.2. Behavioral strategies

Since players change their opponents every round, they always meet a stranger whom they have never met before. For strategic choice, a player relies on the reputation of the opponent (except two unconditional strategies, ALLC and ALLD). Here, we assume the simplest kind of social reputation, binary reputation, as in Nowak and Sigmund (1998b). Each player

Table 1
Sixteen conceivable reputation dynamics that are second-order assessment

C to good	C to bad	D to good	D to bad	Name
G	G	G	G	
G	G	G	B	
G	G	B	G	Simple-standing
G	G	B	B	Scoring
G	B	G	G	
G	B	G	B	
G	B	B	G	Kandori
G	B	B	B	Shunning
B	G	G	G	
B	G	G	B	
B	G	B	G	
B	G	B	B	
B	B	G	G	
B	B	G	B	
B	B	B	G	
B	B	B	B	

has either a good or bad reputation. According to the reputation of the opponents, a player determines their action. Throughout this paper, we assume that individuals know the reputation of the opponent (either correctly or incorrectly; we exclude the possibility that one does not know the reputation of others). There are four possible behavioral strategies, ALLC, ALLD, DISC, and pDISC. An ALLC player always helps the opponent. An ALLD player never gives help. A DISC player helps the good but not the bad (note that "DISC" means a "discriminator"). In contrast, a pDISC (paradoxical-discriminator) helps the bad but not the good. In the present paper, we study three of the four strategies, ALLC, ALLD and DISC, but do not study pDISC strategy because it is odd and not feasible for studying the emergence of cooperation. We assume that players fail to cooperate against their will with small probability ε_e, due to, for example, a lack of resources (Fishman, 2003). We call this "execution error". We do not consider an execution error of the opposite side; players never misimplement intended defection because it is quite unlikely that one accidentally helps others though he intended to do nothing. Later, we see that execution errors play a critical role in our analysis, as in previous studies (Lotem et al., 1999; Panchanathan and Boyd, 2003, 2004).

2.3. *Reputation dynamics of second-order assessment*

Regarding how to judge what action is good and what is bad, we assume that all members in a society share the same norm for moral judgment. Following Ohtsuki and Iwasa (2004), we call this norm "reputation dynamics" in the population. It is denoted by d. In this paper we consider reputation dynamics that are second-order assessment (Brandt and Sigmund, 2005). In order to attach a reputation to a player, an observer must know *what action* the focal player took *to whom*. There are four possible outcomes: (1) the focal player cooperated with a good opponent, (2) he cooperated with a bad opponent, (3) he defected against a good opponent, and (4) he defected against a bad opponent. To each of the four scenarios, the reputation dynamics assigns a reputation, either good or bad. Hence, we have $2^4 = 16$ different reputation dynamics in total, see Table 1.

A player's reputation is updated as follows. Initially ($t = 0$) everyone is supposed to have a good reputation. At round t a focal player randomly finds an opponent and plays a one-shot Prisoner's Dilemma game with him. Based on the focal player's action in the game and the opponent's reputation, a new reputation at round $t + 1$ is assigned to the focal player by reputation dynamics d.

3. Method

We investigate 16 different reputation dynamics one by one. Let us consider one of the reputation dynamics, d. Under this norm, we study evolutionary dynamics of three behavioral strategies, ALLC, ALLD and DISC. In the following, we identify each of these strategies by an integer i: 1 = ALLC, 2 = ALLD, and 3 = DISC. Let x_i be the relative abundance of strategy i, and let W_i be the total payoff of i.

We adopt the following imitation update rule for strategies. A player is sometimes given an opportunity to change his strategy. He randomly samples a player and calculates the difference in payoffs of the two. If a sampled player has a greater payoff then the sampling player imitates the sampled player's strategy with probability proportional to the difference in payoffs. Otherwise a sampling player remains the same strategy. This microscopic updating yields the evolutionary dynamics of the frequencies of strategies, which are called replicator dynamics (Taylor and Jonker, 1978, Hofbauer and Sigmund, 1998), given as follows:

$$\dot{x}_i = x_i(W_i - \bar{W}), \tag{1}$$

where $\bar{W}$ is the average payoff in the entire population, defined as $\bar{W} = x_1W_1 + x_2W_2 + x_3W_3$. This differential equation is defined on the simplex $S_3 = \{(x_1, x_2, x_3)|x_1 + x_2 + x_3 = 1, x_i \geqslant 0\}$. Each corner of the simplex is an equilibrium of the dynamics corresponding to a monomorphic population. Note that in this dynamics only the relative size of payoff matters: additive shifts in payoffs do not alter the dynamics at all.

Throughout our analysis, we assume that the execution error rate ε_e is very small.

4. Results under public reputation

In this section, we assume that each individual knows the correct reputation of others. In this sense the reputation is public information. That is, all individuals have the same opinion on the focal player.

4.1. When $\omega b < c$ holds

When $\omega b < c$ holds, we can prove that ALLD always gains the largest payoff among the three strategies under any conceivable norms. This is plausible because the cost of cooperation c exceeds the maximum return in the next round ωb. In this case, cooperation can never be advantageous. Therefore, in the following we consider the case in which ωb is larger than c unless otherwise specified.

4.2. Scoring

Consider the reputation dynamics called scoring (given as "GGBB" in Table 1) (Brandt and Sigmund, 2004, Ohtsuki and Iwasa, 2004). Under this norm, cooperation is always good and defection is always bad, irrespective of the reputation of the opponent.

By adding the same constant we can make W_2 equal to zero without loss of generality (see Section 3). Let $\tilde{W}_i$ be the total payoff of strategy i after this normalization. Exact calculation in Appendices A.1 and B.1 shows $\tilde{W}_2 = 0$ and

$$\tilde{W}_1 = (1-\varepsilon_e)\frac{(1-\varepsilon_e)\omega b x_3 - c}{1-\omega}, \quad \tilde{W}_3 = \frac{1-\omega+(1-\varepsilon_e)\omega x_1}{1-(1-\varepsilon_e)\omega x_3}\tilde{W}_1. \tag{2}$$

The phase portrait of the dynamics is described in Fig. 1a. In the absence of defectors ($x_2 = 0$) we obtain

$$\tilde{W}_3 - \tilde{W}_1 = -(1-\varepsilon_e)\varepsilon_e\omega\frac{(1-\varepsilon_e)\omega b x_3 - c}{(1-\omega)\{1-(1-\varepsilon_e)\omega x_3\}}. \tag{3}$$

Therefore, there is an equilibrium P on the ALLC–DISC edge, which is stable along this edge. Note, however, that it is not asymptotically stable. On the ALLD–DISC edge, there is an unstable equilibrium Q. There is a line of equilibria located in the center of the simplex, at $x_3 = c/(1-\varepsilon_e)\omega b$. This line always connects the two equilibria, P and Q, irrespective of error rates ε_e, so along this line neutral drift can drive the population away from P.

A small segment on the P–Q line in the vicinity of P (in gray in Fig. 1a) is transversally stable so that any perturbations away from this segment are counterattacked by the dynamics (Brandt and Sigmund, 2006). Therefore, each point on the segment including P is Lyapunov stable. However, the length of this segment is small and of order ε_e. Therefore, the stability of P is quite vulnerable especially when the error rate is small. The other part of the P–Q line (in white in Fig. 1a; it can be quite large when is ε_e is small) is transversally unstable. Any perturbations to increase DISC players are amplified, ultimately leading back to the equilibrium P (see Fig. 1a). On the other hand, any perturbations to decrease DISC players lead to the fixation of ALLD players (see Fig. 1a). Taking neutral drift and small errors into account, we conclude that ALLD is the unique end point of the dynamics after a long run.

4.3. Simple standing

Next, we study the norm given by GGBG in Table 1. This is similar to the "standing" (Sugden, 1986, Leimar and Hammerstein, 2001) for the third-order assessment problem, but is not exactly the same. Hence, we call this "simple-standing". This reputation dynamics differs from scoring in that those who refused to help the bad are regarded as good. This norm has the concept of justified defection (Nowak and Sigmund, 2005).

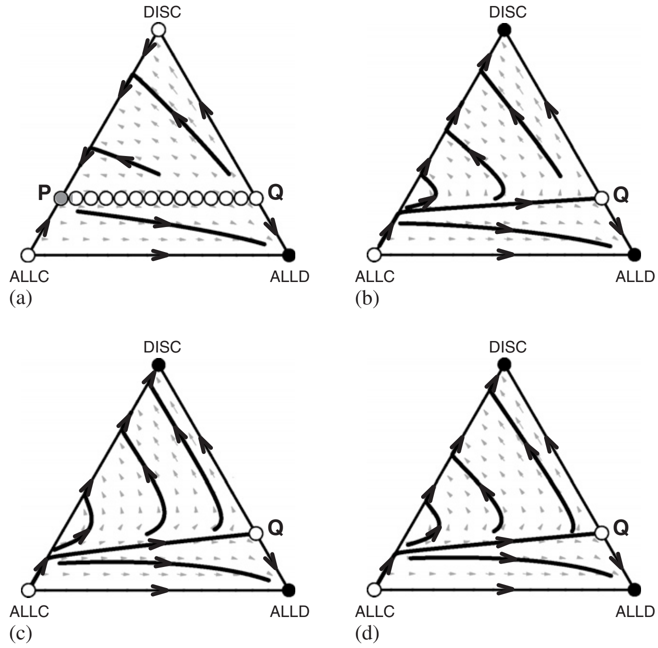

Fig. 1. The phase portrait of evolutionary dynamics of three behavioral strategies, ALLC, ALLD, and DISC when execution errors exist. A triangle represents the phase space, simplex S_3. Each corner of the simplex corresponds to a monomorphic population. Solid circles, circles in gray, and open circles represent asymptotically stable equilibria, Lyapunov stable but not asymptotically stable equilibria, and asymptotically unstable equilibria, respectively. We used $b = 10$, $c = 1$, $w = 0.4$ and $\varepsilon_e = 0.01$. (a) Under scoring: There is a line that consists of equilibria, which connects two equilbria on the edges, P and Q. A small segment on this line in the very vicinity of P (in gray) is transversally stable, but the other part (in white) is transversally unstable. With our parameters the former occupies 0.7% and the latter occupies 99.3% of the P–Q line. P is Lyapunov stable but not asymptotically stable. Along the ALLC–DISC edge, the population eventually reaches the equilibrium P. From P a neutral drift can replace some ALLC players with ALLD players so that the population reaches a transversally unstable part of the P–Q line (in white), where any deviations to decrease DISC players lead to the fixation of ALLD. ALLD is the unique asymptotically stable equilibrium. (b) Under simple-standing: On the ALLC–DISC edge DISC always earns larger payoff than ALLC, so DISC eventually becomes dominant. The corners of DISC and ALLD are both stable equilibria, suggesting that both strategies are evolutionarily stable. The DISC–ALLD edge exhibits bistability and has an unstable equilibrium Q. Its stable manifold is the separatrix dividing the phase space into two basins of attraction of DISC or ALLD. Note that the separatrix is very close to the ALLC–DISC edge near the ALLC corner. Under (c) Kandori and (d) shunning: The phase portrait is qualitatively similar to that of simple standing. DISC is evolutionarily stable.

After the normalization of $\tilde{W}_2 = 0$, we obtain

$$\tilde{W}_1 = (1-\varepsilon_e)\frac{(1-\varepsilon_e)\omega b x_3 - [1+\omega\{1-(1-\varepsilon_e)(x_1+x_3)\}]c}{(1-\omega)[1+\omega\{1-(1-\varepsilon_e)(x_1+x_3)\}]},$$

$$\tilde{W}_3 = (1-\varepsilon_e)\frac{(1-\varepsilon_e)\omega b x_3 - c}{(1-\omega)[1+\omega\{1-(1-\varepsilon_e)(x_1+x_3)\}]} \tag{4}$$

(see Appendix B.2). The phase portrait of this dynamics is given in Fig. 1b. In contrast to scoring, DISC is always favored on the ALLC–DISC edge, because in the absence of defectors,

$$\tilde{W}_3 - \tilde{W}_1 = \frac{\omega\varepsilon_e c}{1-\omega} + O(\varepsilon_e^2) \tag{5}$$

is always positive. The ALLD–DISC edge shows bistability and has an unstable equilibrium Q. The corners of DISC and ALLD are both asymptotically stable equilibria of the dynamics, suggesting they are ESSs. The path that converges to Q (i.e. the stable manifold of Q) runs horizontally and divides the phase space into two regions, and it is a separatrix of the evolutionary dynamics. Above the separatrix is the basin of attraction of DISC. Below that is that of ALLD. Hence, depending on the initial condition, cooperation can be stably maintained by DISC strategy under simple-standing. When the separatrix comes close to the ALLC–DISC edge, it runs downward in the neighborhood of ALLC–DISC edge, and finally converges to ALLC-corner.

4.4. Kandori (GBBG)

We consider the norm represented as GBBG in Table 1. This is the same norm that was proved to be able to maintain the cooperative equilibrium under a much wider condition by Kandori's (1992) classical work. After this, we call this reputation dynamics "Kandori". This norm has the concept of justified defection. In addition helping a bad player is a bad action. Too much generosity is regarded bad (Takagi, 1996) under this norm.

The phase portrait of the evolutionary dynamics under this norm is given in Fig. 1c (see Appendix B.3 for calculation of payoffs). It is qualitatively the same as that in simple-standing. Again DISC is always favored against ALLC on the ALLC–DISC edge. Both DISC and ALLD are ESSs. The path that converges to the unstable Q on the ALLD–DISC edge is the separatrix of the dynamics. It lies between two basins of attraction of ALLD or DISC. We conclude that the norm Kandori can also foster sustained cooperation.

4.5. Shunning (GBBB)

We consider the reputation dynamics given as GBBB in Table 1, called shunning (Nowak and Sigmund, 2005). Shunning is a strict norm in such a sense that those who interacted with bad players are immediately labeled as bad, irrespective of their action (C or D). Under this norm, a player gains a good reputation only by cooperating with a good player.

The phase portrait of the evolutionary dynamics of strategies under this norm is given in Fig. 1d (see Appendix B.4 for calculation of payoffs). It is qualitatively the same as simple-standing and Kandori. Shunning enables sustained cooperation, too.

4.6. Other reputation dynamics

We have studied four reputation dynamics out of the 16 in Table 1. Under each of the other 12 reputation dynamics, we can prove that ALLD is the unique global attractor. The proof is in Appendix B.5. Hence, they cannot foster cooperation.

5. With private reputation errors

So far we have studied evolutionary dynamics of strategies under the assumption of public information. In this section, we slightly loosen this assumption and study the effect of small amount of private reputation errors. Suppose that at each round t, a player has an incorrect opinion on a focal player with small probability ε_p due to some reasons. We consider errors of both directions: a player may mistakenly regard a good player as bad or he may mistakenly regard a bad one as good. For example, when player X's correct reputation is good, a vast majority regards X as a good person but a small amount of individuals who have committed this error think that X is a bad person. The question is how the reputation of player Y, who is the opponent of X in the next round, is determined, because it is dependent on X's reputation but some think X good and others think X bad. Here, we assume that reputation is determined publicly in every round: that is, Y's new reputation is first determined by the majority rule in the society and everyone shares this new information. After this public consensus is reached, each individual may again independently deviate from this due to private reputation errors in the next round. As a result, private reputation errors committed in a round in evaluating other members are not carried over to the following round. We discuss the appropriateness of this assumption later.

Thus, we will consider two different sorts of errors, execution errors (with the rate ε_e) and private reputation errors (ε_p). We assume that ε_e and ε_p are very small. Notice that the following results are derived for $\omega b > c$.

5.1. Scoring

Consider scoring (see Table 1). Introduction of private reputation errors does not change the structure of evolutionary dynamics qualitatively. See Fig. 2a. We have an equilibrium P on the ALLC–DISC edge. It is stable along this edge but is not asymptotically stable. On the ALLD–DISC edge there is an unstable equilibrium Q. The straight line connecting P and Q consist of equilibria (see Appendix B.1). A small segment on this line in the very vicinity of P (in gray in Fig. 2a) is transversally stable, which length is of order $\sqrt{\varepsilon_e^2 + \varepsilon_p^2}$. The other part (in white in Fig. 2a) is transversally unstable.

Even if the initial population has plenty of DISC players, the equilibrium P with a mixture of DISC and ALLC is eventually reached. A neutral drift can replace ALLC players with ALLD players along the line of equilibria. Once the number of ALLD players exceeds a certain amount and the population reaches the unstable part of the segment (in white in Fig. 2a), a small deviation to decrease DISC players leads to the fixation of unconditional defectors. Hence, scoring is unlikely to be able to maintain stable cooperation. See Appendix B.1 for detailed calculations.

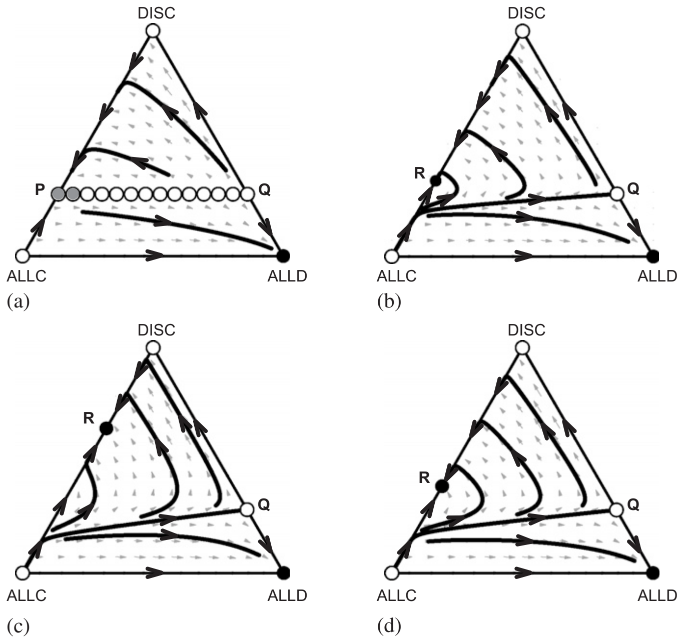

Fig. 2. The phase portrait of evolutionary dynamics of three strategies, ALLC, ALLD, and DISC, when both execution and private reputation errors exist. The meanings of symbols used are the same as in Fig. 1. We used $b = 10$, $c = 1$, $w = 0.4$, $\varepsilon_e = 0.01$ and $\varepsilon_p = 0.04$. (a) Under scoring: The phase portrait is qualitatively unchanged from Fig. 1a. There is a line of equilibria in the center of the simplex. About 6% of the line in the vicinity of P is transversally stable (in gray) in our parameters while the rest 94% (in white) is transversally unstable. P is Lyapunov stable but not asymptotically stable. After some neutral drifts along the P–Q line, a small deviation to decrease DISC players leads to the fixation of ALLD. (b) Under simple-standing: Unlike Fig. 1b, the ALLC–DISC edge exhibits bistability. There is a coexistence equilibrium R between ALLC and DISC that is stable along the ALLC–DISC edge. The separatrix, the path converging to unstable equilibrium Q, divides the phase space into two regions. Note that the separatrix is very close to the ALLC–DISC edge around the ALLC corner. The equilibrium R lies in the upper region, so it is asymptotically stable. Hence, stable coexistence of unconditional and conditional cooperators is realized. Under (c) Kandori and (d) shunning: The phase portrait is similar to that of simple standing. The coexistence equilibrium R is asymptotically stable, hence an attractor of the dynamics.

5.2. Simple-standing, Kandori, amd shunning

Under simple-standing, Kandori, or shunning, DISC strategy is evolutionarily stable in the absence of private reputation errors. However, the dynamics of behavioral strategies can change if small private reputation errors are included in addition to small execution errors.

As an example, consider simple-standing (see Table 1). Fig. 2b shows the evolutionary dynamics of three strategies under simple-standing (for detailed calculations of payoffs see Appendix B.2). We see that the introduction of private reputation errors alters the dynamics on the ALLC–DISC edge. In fact Eq. (5) changes to

$$\tilde{W}_3 - \tilde{W}_1 = \frac{-\omega b x_3 \varepsilon_p + \{\omega\varepsilon_e + (1+\omega x_3)\varepsilon_p\}c}{1-\omega} + O(\varepsilon^2), \tag{6}$$

where $O(\varepsilon^2)$ represents small terms of magnitude of ${\varepsilon_e}^2$ and ${\varepsilon_p}^2$. If

$$\frac{b}{c} > \frac{1}{\omega} + \frac{1}{\varepsilon_p/(\varepsilon_e+\varepsilon_p)}, \tag{7}$$

holds (the inequality (7) is satisfied often when ε_p is large in comparison to ε_e), then we have a stable equilibrium R along the ALLC–DISC edge. We stress that DISC is no more evolutionarily stable. On the ALLD–DISC edge, there is an unstable equilibrium Q. The path that converges to Q is the separatrix of the dynamics, dividing the phase space into two regions. The lower region is the basin of attraction of ALLD. Remarkably, all the internal points of the ALLC–DISC edge including R belong to the upper region (Fig. 2b). Therefore, R is an attractor of the dynamics: from any initial states in the region above the separatrix, the population converges to R. R is a polymorphic equilibrium in which unconditional and conditional cooperators coexist, and it is stable against the invasion of ALLD. R is located away from the separatrix.

If private reputation errors are not frequent, Eq. (7) is not satisfied, and DISC is favored against ALLC on the ALLC–DISC edge. DISC remains evolutionarily stable as in Fig. 1a (see Appendix B.2 for further details), as long as $\omega b > c$ holds. In contrast, if private reputation errors are sufficiently frequent compared with execution errors, DISC is susceptible to those errors while ALLC is not, because ALLC strategists do not use the reputation of others at all. This brings an advantage to unconditional cooperators, leading to the coexistence.

Qualitatively similar results hold for Kandori and shunning. See Figs. 2c–d. For both norms we have a stable coexistence equilibrium R on the ALLC–DISC edge if private reputation errors occur. In Appendices B.3–4 we show the conditions under which the coexistence equilibrium appears.

5.3. Other reputation dynamics

None of the other 12 reputation dynamics (see Table 1) can realize sustained cooperation in the absence of private reputation errors, and this conclusion remains the same when there are private reputation errors. See Appendix B.5 for the proof.

6. Selection pressure to reduce private reputation errors

In the last section, we assumed that reputation was determined publicly but each member has a chance to incorrectly memorize the reputation of a focal person. Those who committed a private reputation error and had an incorrect opinion toward a focal person can modify his error through communicating with other members of the population. In this section, we will show that there is a selective pressure at work for each player to reduce his own private reputation error rate ε_p.

Consider a mutant of DISC players, called DISC' player, who also uses DISC as behavioral strategy and cooperates with good opponents only. A DISC' player communicates with others concerning the reputation of the opponent and attempts to adjust his opinion to sympathize with the majority if his private opinion on the opponent differs from that of the majority. Let ε'_p be the probability that he has an incorrect opinion on a focal player. From the nature of DISC' strategy we expect $\varepsilon'_p < \varepsilon_p$, because communication enables a DISC' player to modify his incorrect opinion if any. Suppose that the population consists of the fraction of x_3^* of DISC players and that of $1 - x_3^*$ of ALLC players. Let W_4 be the total payoff of rare DISC' players in that population. Under simple standing, Kandori, or shunning, we obtain

$$W_4 - W_3 \approx (\varepsilon_p - \varepsilon'_p)\frac{\omega b x_3^* - c}{1-\omega}. \tag{8}$$

We can prove that Eq. (8) is always positive at the coexistence equilibrium of ALLC and DISC (represented as R in Figs. 2b–d). This implies that selection favors DISC' mutants more than wild-type DISC players. In other words, the ability to correct private reputation errors is favored by natural selection. In order to perform well in a society where the indirect reciprocity operates, players should care not about "how *I* think" but about "how *others* think". Through communication, players correct private reputation errors they committed and side with the majority. We expect that, as a result of this selection, ε_p is kept small if communication among members is not very costly.

7. Discussion

We have studied evolutionary dynamics of three strategies, ALLC, ALLD, and DISC, under 16 possible reputation dynamics that are based on second-order assessment (Brandt and Sigmund, 2005). This is the first study that has systematically explored global evolutionary dynamics for all conceivable norms. As a result, we found that only the three norms out of 16, simple-standing, Kandori, and shunning (see Table 1) could realize sustained cooperation while the other 13, including scoring, could not. First, we considered the effect of execution errors only. Under the three norms, the corner of DISC strategy was asymptotically stable equilibrium of the dynamics; hence, DISC strategy was evolutionarily stable. Second, we incorporated private reputation errors of evaluating others. We obtained the stable coexistence of unconditional and conditional cooperators. Finally, we discussed natural selection favoring players with smaller private reputation errors, who communicate with others and sympathize with the majority in opinion.

As the benefit-to-cost ratio of cooperation b/c changes, the evolutionary dynamics change in the following manner if simple standing, Kandori, or shunning is adopted. When small private reputation errors exist and the benefit-cost ratio of cooperation b/c is large, we obtain the stable coexistence of unconditional and conditional cooperators (equilibrium R in Figs. 2b–d). As b/c ratio becomes smaller, conditional cooperators become more abundant at the equilibrium: in the phase space in Figs. 2b–d the coexistence equilibrium R approaches the DISC-corner, until finally unconditional cooperators disappear and a monomorphic population of conditional cooperators realizes sustained cooperation as an ESS, as in Figs. 1b–d. As b/c decreases further, the unstable equilibrium Q and the separatrix in Figs. 1b–d move upward and the basin of attraction of ALLD expands. When b/c becomes less than $1/\omega$, the basin of attraction of DISC vanishes and ALLD prevails from any initial conditions.

Hence, $b/c > 1/\omega$ is the condition for the evolution of cooperation by indirect reciprocity. If this is satisfied, then there are two situations with respect to the composition of the population. For example under simple-standing norm, cooperation is sustained by the mixture of unconditional and conditional cooperators if $b/c > 1/\omega + [\varepsilon_p/(\varepsilon_e + \varepsilon_e)]^{-1}$ holds; but cooperation is sustained by a monomorphic population of conditional cooperators if instead $1/\omega < b/c < 1/\omega[\varepsilon_p/(\varepsilon_e + \varepsilon_p)]^{-1}$ holds. In the absence of private reputation errors, we always obtain the latter because the term $[\varepsilon_p/(\varepsilon_e + \varepsilon_p)]^{-1}$is infinitely large.

7.1. Leading two and shunning

Ohtsuki and Iwasa (2004) studied 256 reputation dynamics that are third-order assessment. A norm that is third-order assessment judges an observed action by (i) the action of the focal player, (ii) the reputation of the opponent, and (iii) the reputation of the focal player. As a result of ESS analysis, Ohtsuki and Iwasa found eight combinations of a reputation dynamics and a behavioral strategy, called the leading eight. The reputation dynamics of two of the leading eight do not use (iii), and they are essentially based on second-order assessment. Those two norms are simple-standing and Kandori, and may be called the "leading two" in the indirect reciprocity of second-order assessment. We note that the leading two has the concept of justified defection; defection against a bad person is regarded as a good behavior (see Table 1). This is quite effective in expelling cheaters when they are rare. However, it is also true that defectors can gain a good reputation without giving if they are abundant in the population, which potentially weakens the cooperative strategies.

It is noteworthy that shunning can make DISC-strategy evolutionarily stable, although it does not belong to the leading eight, hence nor to the leading two. This difference comes from different assumptions on the initial condition. In this paper, we assumed that everyone has a good reputation at the start of each generation. In contrast, in Ohtsuki and Iwasa (2004, 2006), the initial fraction of good persons can be an intermediate value, and the fraction of good persons in the leading eight automatically increases and becomes close to unity after multiple rounds of the game. For the population adopting shunning, the fraction of good players at the ESS monotonically declines with time, but the rate of decline is slow if everyone is good in the initial population and if error rate is very small. When this is the case, the repeated game terminates far earlier before many players become bad in the population, and cooperation is sustained under shunning, which is in agreement with that of Takahashi and Mashima (2003).

7.2. Kinds of errors and the evolutionary outcome

When private reputation errors occur relatively more frequently than execution errors, the stable coexistence of unconditional and cooperators is likely to be achieved. This indicates that different types of errors have different impacts on the evolutionary dynamics, even though they occur rarely. Since the difference in payoffs between ALLC and DISC strategies results from nothing but errors, the dynamics are highly sensitive to the manner in which errors are introduced into models. The presence of people who are labeled bad favors DISC because ALLC misses an opportunity to defect without being punished. Hence, a large execution error ε_e favors DISC over ALLC. In contrast, larger private reputation errors of evaluation of others ε_p would jeopardize DISC. If the reputation is incorrect, a DISC player defects against a "bad" opponent, and may find out being punished by others, because the opponent was in fact considered as "good" by other members.

To explore this further, we introduce the third error, called "reporting error", into our model and examine its effect (Ohtsuki and Iwasa, 2004). The action of a focal player is observed by a few others, who report it to the rest of the population, and then a collective decision on the reputation in the next round is made (this is called "indirect observation model" by Ohtsuki and Iwasa (2004)). A reporting error occurs in this process: a reporter may mistakenly report the wrong information about the action of the focal player to all the others (note that we do not study intentional lying; for studies on lying see Nakamaru and Kawata, 2004). Therefore, the reporting errors influence all members in the population.

As an example consider simple-standing. We consider three errors; execution errors, reporting errors, and private reputation errors, with rates ε_e, ε_r and ε_p, respectively. Note that the former two affect all players but the latter one influences the error-committer only. Private reputation errors ε_p disfavor conditional cooperation by indirect reciprocity. If players have different opinions on the same person, errors undermine cooperation (Takahashi and Mashima, 2003; Brandt and Sigmund, 2005). In contrast, reporting errors would not harm conditional cooperators because those errors cause incorrect evaluation on a member by all the players in the population. On the contrary, reporting errors ε_r increase the fraction of player labeled "bad" by public, and hence favor DISC over ALLC, in the same way as execution errors ε_e. Preliminary calculation shows that reporting errors only changes ε_e in Eq. (7) to $\varepsilon_e + \varepsilon_r$.

7.3. Advantage of adopting evaluation of the majority

We have shown that players gain larger payoff by tuning their private (incorrect) opinion to that of the public if they differ. In order to receive help from others, it is very important to be regarded as good by others. What really matters in indirect reciprocation is not "how *I* think of me" but "how *others* think of me", because it is from others that a player receives help. Therefore, each player corrects private reputation errors through communication with others. We believe that in this process language capability of humans must play an important role (Fehr and Fischbacher, 2003). When the effect of private reputation errors is kept small such elaborated norms as simple-standing, Kandori, or shunning, are able to contribute much to sustained cooperation. Based on the analysis of the present paper, we conjecture that the evolution of language have caused a rapid evolution of indirect reciprocity. Linguistic communication enables humans to construct public information that all members sympathize with. As a result, reputation realizes community enforcement for cooperation.

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Appendix A. Calculation of total payoff W_i

Let W_i be the discounted total payoff of strategy i ($1 = \text{ALLC}$, $2 = \text{ALLD}$, and $3 = \text{DISC}$). Let $P_i(t)$ ($t = 1, 2, \ldots$) be the average payoff of strategy i at round t. Then W_i is written as

$$W_i = \sum_{t=1}^{\infty} \omega^{t-1} P_i(t), \tag{A.1}$$

where ω is a discount factor ($0 \leqslant \omega < 1$). Let $G_i(t)$ ($t = 0, 1, 2, \ldots$) be the fraction of individuals among i strategists whose (correct) reputation is good at the end of round t. The fraction of bad individuals among i strategists is given by $B_i(t) = 1 - G_i(t)$. At round t, a player receives cooperation (i) if he meets an ALLC player or (ii) if he meets a DISC player by whom he is thought to be a good player. Regarding when one pays cost, an ALLC player always cooperates with others while a DISC player cooperates only when he meets a good person. Taking those into consideration, we obtain

$$\begin{aligned}
P_1(t) &= (1-\varepsilon_e)[\{x_1 + \{(1-\varepsilon_p)G_1(t-1) + \varepsilon_p B_1(t-1)\}x_3\}b - c],\\
P_2(t) &= (1-\varepsilon_e)[\{x_1 + \{(1-\varepsilon_p)G_2(t-1) + \varepsilon_p B_2(t-1)\}x_3\}b],\\
P_3(t) &= (1-\varepsilon_e)[\{x_1 + \{(1-\varepsilon_p)G_3(t-1) + \varepsilon_p B_3(t-1)\}x_3\}b - \{(1-\varepsilon_p)G(t-1) + \varepsilon_p B(t-1)\}c].
\end{aligned} \tag{A.2}$$

Here, $G(t) \equiv x_1 G_1(t) + x_2 G_2(t) + x_3 G_3(t)$ and $B(t) \equiv x_1 B_1(t) + x_2 B_2(t) + x_3 B_3(t)$. ε_e and ε_p are error rates of execution error and private reputation error respectively. From Eqs. (A.1) and (A.2) we need to know $G_i(t)$ ($t = 0, 1, 2, \ldots$) in order to derive W_i.

From our assumption, we have $G_1(0) = G_2(0) = G_3(0) = 1$ as an initial condition. Let us derive a recursion on $G_i(t)$. Whether a player is assigned a good or bad reputation depends on *what action* he takes *to whom*. Consider the interaction at round t.

(1) An ALLC player cooperates with a good player with probability $m_{11} = (1-\varepsilon_e)G(t-1)$, cooperates with a bad player with probability $m_{12} = (1-\varepsilon_e)B(t-1)$, defects against a good player with probability $m_{13} = \varepsilon_e G(t-1)$, and defects against a bad player with probability $m_{14} = \varepsilon_e B(t-1)$.
(2) An ALLD player cooperates with a good player with probability $m_{21} = 0$, cooperates with a bad player with probability $m_{22} = 0$, defects against a good player with probability $m_{23} = G(t-1)$, and defects against a bad player with probability $m_{24} = B(t-1)$.
(3) A DISC player cooperates with a good player with probability $m_{31} = (1-\varepsilon_e)(1-\varepsilon_p)G(t-1)$, cooperates with a bad player with probability $m_{32} = (1-\varepsilon_e)\varepsilon_p B(t-1)$, defects against a good player with probability $m_{33} = \{1-(1-\varepsilon_e)(1-\varepsilon_p)\}G(t-1)$, and defects against a bad player with probability $m_{34} = \{1-(1-\varepsilon_e)\varepsilon_p\}B(t-1)$.

Now define a 3×4 matrix M as

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{pmatrix}. \tag{A.3}$$

The components m_{ij} are linear functions of $G(t-1)$, so we should write M as $M[G(t-1)]$. Also define a 4×1 "reputation dynamics" matrix (or a column vector) D as

$$D = (\, D_{GC} \quad D_{BC} \quad D_{GD} \quad D_{BD} \,)^{\mathrm{T}}. \tag{A.4}$$

Here each of D_{GC}, D_{BC}, D_{GD}, D_{BD} corresponds to "C to *good*", "C to *bad*", "D to *good*", "*D* to *bad*". If the focal reputation dynamics d assigns a good reputation in this situation then the corresponding D-value is 1, otherwise it is 0. For example, scoring is (C to *good*, C to *bad*, D to *good*, D to *bad*) = (G, G, B, B) so it yields $D = (1 \quad 1 \quad 0 \quad 0)^{\mathrm{T}}$. Using those notations above we obtain the following recursion on $G_i(t)$'s:

$$(G_1(t) \quad G_2(t) \quad G_3(t))^{\mathrm{T}} = M[G(t-1)]D. \tag{A.5}$$

Since $G(t) \equiv x_1G_1(t) + x_2G_2(t) + x_3G_3(t)$, we have $G(t) = (x_1 \quad x_2 \quad x_3) \cdot M[G(t-1)]D$. This is a linear recursion on $G(t)$, so we are able to solve that. Then we can solve Eq. (A.5), so we obtain W_i from Eqs. (A.1) and (A.2).

Appendix B. Results for each of 16 norms

In the following, we use notation like "GGBB" to represent a norm out of 16. For example, GGBB means (C to *good*, C to *bad*, D to *good*, D to *bad*) = (G, G, B, B), so it represents scoring (see also Table 1). Let W_i be the discounted total payoff of strategy i, calculated in the previous section. By subtracting W_2 from each W_i we can normalize payoff such that $\tilde{W}_1 = W_1 - W_2$, $\tilde{W}_2 = W_2 - W_2 = 0$, and $\tilde{W}_3 = W_3 - W_2$. We assume that the magnitude of errors $\varepsilon \equiv \sqrt{\varepsilon_e^2 + \varepsilon_p^2}$ is so small.

B.1. GGBB (scoring)

From Appendix A we obtain

$$\tilde{W}_1 = (1-\varepsilon_e)\frac{(1-\varepsilon_e)(1-2\varepsilon_p)\omega bx_3 - c}{1-\omega},$$

$$\begin{aligned}\tilde{W}_3 &= (1-\varepsilon_e)\frac{(1-\varepsilon_e)(1-2\varepsilon_p)\omega bx_3 - c}{1-\omega}\frac{1-\omega-(1-2\omega)\varepsilon_p + (1-\varepsilon_e)(1-2\varepsilon_p)\omega x_1}{1-(1-\varepsilon_e)(1-2\varepsilon_p)\omega x_3} \\ &= \tilde{W}_1\frac{1-\omega-(1-2\omega)\varepsilon_p + (1-\varepsilon_e)(1-2\varepsilon_p)\omega x_1}{1-(1-\varepsilon_e)(1-2\varepsilon_p)\omega x_3}.\end{aligned} \tag{B.1}$$

When $x_2 = 0$,

$$\tilde{W}_3 - \tilde{W}_1 = -(1-\varepsilon_e)\frac{\{\varepsilon_p + \varepsilon_e(1-2\varepsilon_p)\omega\}\{(1-\varepsilon_e)(1-2\varepsilon_p)\omega bx_3 - c\}}{(1-\omega)\{1-(1-\varepsilon_e)(1-2\varepsilon_p)\omega x_3\}}. \tag{B.2}$$

From these, two equilibria, P and Q, and the line of equilibria are at $x_3 = c/(1-\varepsilon_e)(1-2\varepsilon_p)\omega b$.

B.2. GGBG (simple-standing)

From Appendix A, we obtain

$$\tilde{W}_1 = (1-\varepsilon_e)\frac{(1-\varepsilon_e)(1-2\varepsilon_p)\omega bx_3 - [1+\omega\{1-(1-\varepsilon_e)(x_1+(1-\varepsilon_p)x_3)\}]c}{(1-\omega)[1+\omega\{1-(1-\varepsilon_e)(x_1+(1-\varepsilon_p)x_3)\}]},$$

$$\tilde{W}_3 = (1-\varepsilon_e)\frac{(1-\varepsilon_e)(1-\varepsilon_p)(1-2\varepsilon_p)\omega bx_3 - [1-\varepsilon_p\{1-\omega(1-(1-\varepsilon_e)(x_1+(1-\varepsilon_p)x_3))\}]c}{(1-\omega)[1+\omega\{1-(1-\varepsilon_e)(x_1+(1-\varepsilon_p)x_3)\}]}, \tag{B.3}$$

When $x_2 = 0$,

$$\tilde{W}_3 - \tilde{W}_1 = \frac{-\omega bx_3\varepsilon_p + \{\omega\varepsilon_e + (1+\omega x_3)\varepsilon_p\}c}{1-\omega} + O(\varepsilon^2). \tag{B.4}$$

First, in the absence of defectors we have

$$\tilde{W}_3 - \tilde{W}_1 = \frac{\omega\varepsilon_e c}{1-\omega} + O(\varepsilon_e^2), \tag{B.5}$$

which is always positive. Therefore, DISC is always favored against ALLC on the ALLC–DISC edge. Second consider when private reputation errors exist. If

$$\frac{b}{c} > \frac{1}{\omega} + \frac{1}{\alpha}, \quad \left(\alpha \equiv \varepsilon_p/(\varepsilon_e + \varepsilon_p)\right), \tag{B.6}$$

is satisfied then the coexistence equilibrium R emerges on the ALLC–DISC edge as in Fig. 2b. If not, DISC is evolutionarily stable as in Fig. 1a as long as $\omega b > c$.

B.3. GBBG (Kandori)

From Appendix A we obtain

$$\tilde{W}_1 = (1-\varepsilon_e) \times \frac{(1-\varepsilon_e)(1-2\varepsilon_p)\omega b x_3[1+\omega\{-1+(1-\varepsilon_e)(1-2\varepsilon_p)x_3\}] - [1+\omega\{1-(2x_1+x_3)(1-\varepsilon_e)\}]c}{(1-\omega)[1+\omega\{\varepsilon_e-(1-\varepsilon_e)(x_1-x_2)\}]},$$

$$\tilde{W}_3 = (1-\varepsilon_e) \times \frac{(1-\varepsilon_e)(1-2\varepsilon_p)\omega b x_3\{1-\omega(1-\varepsilon_e)(1-2\varepsilon_p)x_1-(1+\omega)\varepsilon_p\} - [(1-\varepsilon_p)\{(1-\omega)-2\omega(1-\varepsilon_e)\varepsilon_p x_3\}+\omega\{1-(1-\varepsilon_e)x_1\}]c}{(1-\omega)[1+\omega\{\varepsilon_e-(1-\varepsilon_e)(x_1-x_2)\}]} \tag{B.7}$$

When $x_2 = 0$,

$$\tilde{W}_3 - \tilde{W}_1 = \frac{-\omega b x_3\{-\omega\varepsilon_e + (1-\omega)\varepsilon_p\} + \{\omega\varepsilon_e + (1-\omega+2\omega x_3)\varepsilon_p\}c}{(1-\omega)(1-\omega x_1)} + O(\varepsilon^2). \tag{B.8}$$

Without private reputation errors, Eq. (B.8) is rewritten as

$$\tilde{W}_3 - \tilde{W}_1 = \frac{\omega(\omega b x_3 + c)\varepsilon_e}{(1-\omega)(1-\omega x_1)} + O(\varepsilon_e^2). \tag{B.9}$$

This is always positive, hence DISC is favored on the ALLC–DISC edge. Next, consider when private reputation errors exist. In this case the coexistence equilibrium R exists on the ALLC–DISC edge as in Fig. 2c if

$$\frac{b}{c} > \left(\frac{1}{\omega} + \frac{1}{\alpha}\right)\frac{1}{1-\omega/\alpha} \quad \text{and} \quad \alpha > \omega \quad (\alpha \equiv \varepsilon_p/(\varepsilon_e+\varepsilon_p)), \tag{B.10}$$

is satisfied. If not DISC is evolutionarily stable as in Fig. 1c as long as $\omega b > c$.

B.4. GBBB (shunning)

From Appendix A we obtain

$$\tilde{W}_1 = (1-\varepsilon_e)\frac{(1-\varepsilon_e)(1-2\varepsilon_p)(1-\omega)\omega b x_3 - [1-\omega(1-\varepsilon_e)\{x_1+(1-\varepsilon_p)x_3\}]c}{(1-\omega)[1-\omega(1-\varepsilon_e)\{x_1+(1-\varepsilon_p)x_3\}]},$$

$$\tilde{W}_3 = (1-\varepsilon_e)\frac{(1-\varepsilon_e)(1-\varepsilon_p)(1-2\varepsilon_p)(1-\omega)\omega b x_3 - [1-\omega-\varepsilon_p\{1-\omega(2-(1-\varepsilon_e)(x_1+(1-\varepsilon_p)x_3))\}]c}{(1-\omega)[1-\omega(1-\varepsilon_e)\{x_1+(1-\varepsilon_p)x_3\}]}. \tag{B.11}$$

When $x_2 = 0$,

$$\tilde{W}_3 - \tilde{W}_1 = \frac{-\varepsilon_p(1-\omega)\omega b x_3 + \{\omega\varepsilon_e + (1-\omega+\omega x_3)\varepsilon_p\}c}{(1-\omega)^2} + O(\varepsilon^2). \tag{B.12}$$

Without private reputation errors Eq. (B.12) is rewritten as

$$\tilde{W}_3 - \tilde{W}_1 = \frac{\omega\varepsilon_e c}{(1-\omega)^2} + O(\varepsilon_e^2). \tag{B.13}$$

This is always positive, hence DISC is always favored on the ALLC–DISC edge. With private reputation errors, if

$$\frac{b}{c} > \frac{1}{\omega} + \frac{1}{\alpha(1-\omega)}, \quad (\alpha \equiv \varepsilon_p/(\varepsilon_e+\varepsilon_p)), \tag{B.14}$$

is satisfied then the coexistence equilibrium R emerges on the ALLC–DISC edge as in Fig. 2d. If not, DISC is evolutionarily stable as in Fig. 1d as long as $\omega b > c$.

B.5. Other 12 norms

From Appendix A, we can calculate payoffs of three strategies under each of other 12 norms, see Table 2. In Table 2, we show normalized payoffs of ALLC ($\tilde{W}_1$) and DISC ($\tilde{W}_3$) in the absence of any errors. We note that when errors do not

Table 2
Normalized payoffs $\tilde{W}_1$ and $\tilde{W}_3$ for 12 norms other than four norms explained in the text

Norm	$\tilde{W}_1$	$\tilde{W}_3$
(1) GGGG	$-c/(1-\omega)$	$-c/(1-\omega)$
(2) GGGB	$-c/(1-\omega)$	$-c/(1-\omega)$
(3) GBGG	$-c/(1-\omega)$	$-c/(1-\omega)$
(4) GBGB	$-c/(1-\omega)$	$-c/(1-\omega)$
(5) BGGG	$-((\omega bx_3+c)+\omega c(x_1+x_3))/(1-\omega)\{1+\omega(x_1+x_3)\}$	$-(\omega bx_3+c)/(1-\omega)\{1+\omega(x_1+x_3)\}$
(6) BGGB	$(-(\omega bx_3+c)\{1-\omega(1-x_1)\}+\omega(\omega bx_3-c)(1-x_2))/(1-\omega)\{1+\omega(x_1-x_2)\}$	$-((\omega bx_3+c)\{1-\omega(1-x_1)\})/(1-\omega)\{1+\omega(x_1-x_2)\}$
(7) BGBG	$-c/(1-\omega)$	$-c/(1-\omega^2)$
(8) BGBB	$(-c\{1-\omega(1-x_1)\}+\omega(\omega bx_3-c))/((1-\omega)(1+\omega x_1))$	$-c\{1-\omega(1-x_1)\}/(1-\omega)(1+\omega x_1)$
(9) BBGG	$-(\omega bx_3+c)/(1-\omega)$	$-(\omega bx_3+c)/(1-\omega x_1)/(1-\omega)(1+\omega x_3)$
(10) BBGB	$-((\omega bx_3+c)/(1-\omega x_2))$-$(\omega c(1-x_2))/((1-\omega)(1-\omega x_2))$	$-(\omega bx_3+c)/(1-\omega x_2)$
(11) BBBG	$-(c(1-\omega x_1)+\omega(\omega bx_3+c))/(1-\omega)\{1+\omega(x_2+x_3)\}$	$-(c(1-\omega x_1))/(1-\omega)\{1+\omega(x_2+x_3)\}$
(12) BBBB	$-c/(1-\omega)$	$-c$

For the notation of norms, see Table 1 and Appendix B.

exist, evolutionary dynamics of three strategies are structurally stable for each norm. This means that the introduction of small amount of errors do not change the dynamics qualitatively. Hence, it is enough to analyse the error-free case.

We will prove below that ALLD is a unique global attractor for norms (1)–(12) in Table 2. For norms (1)–(5), (7) and (9)–(12), it is easy to see that W_2 is always the largest among W_i $(i=1,2,3)$, so ALLD is a unique global attractor of the dynamics. For norms (6) and (8), if $x_3 \geqslant c/\omega b$ then W_3 is the smallest of the three, so x_3 decreases such that it becomes below $c/\omega b$. This suggests that the region defined as $\{(x_1, x_2, x_3)|x_1+x_2+x_3=1, x_3<c/\omega b, x_i \geqslant 0\}$ is positively invariant under the replicator dynamics. In this region, W_2 is always largest of the three, so ALLD is a unique global attractor of the dynamics. This ends the proof.

Appendix C. Payoff of DISC' players

Consider DISC' players. They adopt DISC strategy as normal DISC players do. They also communicate with others and try to correct their opinion if it is different from the majority. Let ε'_p be the probability that a DISC' player has an incorrect opinion on a focal player.

We introduce a considerably small amount of DISC' players into the population and calculate the difference in payoff from normal DISC players. Since DISC' players are rare in the population they do not affect the payoff of the other strategies.

We need to conduct almost the same calculation as in Appendix A. Let W_4 be DISC' players' total payoff over rounds. As in Appendix A, it is given by the weighted sum of payoff at the tth round $P_4(t)$ as $W_4=\sum_{t=1}^{\infty}\omega^{t-1}P_4(t)$. Similarly to Eq. (A.2), $P_4(t)$ is given as

$$P_4(t)=(1-\varepsilon_e)[\{x_1+\{(1-\varepsilon_p)G_4(t-1)+\varepsilon_p B_4(t-1)\}x_3\}b-\{(1-\varepsilon'_p)G(t-1)+\varepsilon'_p B(t-1)\}c]. \tag{C.1}$$

Here $G_4(t)$ and $B_4(t)$ represent the fraction of good or bad players among DISC' strategists at the end of round t. As in Eq. (A.5) the recursion on $G_4(t)$ is given by

$$\begin{pmatrix} G_1(t) & G_2(t) & G_3(t) & G_4(t) \end{pmatrix}^{\mathrm{T}} = M[G(t-1)]D. \tag{C.2}$$

Here $M[G(t-1)]=(m_{ij})$ is the 4×4 matrix. m_{ij}'s $(i=1,2,3)$ are defined as in Appendix A. m_{4j} is the same as m_{3j} except that ε_p is substituted by ε'_p. The 4×1 matrix D is given by Eq. (A.4).

From these calculations, we obtain the following results. For simple-standing (GGBG, see Table 1), we have

$$W_4-W_3 \approx (\varepsilon_p-\varepsilon'_p)\frac{\omega bx_3-c(1-\omega x_2)}{(1-\omega)(1+\omega x_2)}. \tag{C.3}$$

For Kandori (GBBG, see Table 1) we have

$$W_4-W_3 \approx \left(\varepsilon_p-\varepsilon'_p\right)\frac{\omega bx_3\{1-\omega(x_1-x_2)\}-c\{1-\omega(x_1+x_2)\}}{(1-\omega)\{1-\omega(x_1-x_2)\}}. \tag{C.4}$$

For shunning (GBBB, see Table 1) we have

$$W_4 - W_3 \approx (\varepsilon_p - \varepsilon'_p)\frac{\omega b x_3(1-\omega) - c\{1-\omega(1+x_2)\}}{(1-\omega)\{1-\omega(1-x_2)\}}, \tag{C.5}$$

By substituting $(x_1, x_2, x_3) = (1 - x_3^*, 0, x_3^*)$ in Eqs. (C.3)–(C.5) we obtain Eq. (8) in the main text.

References

Alexander, R.D., 1987. The Biology of Moral Systems. Aldine de Gruyter, New York.

Bolton, G., Katok, E., Ockenfels, A., 2005. A cooperation among strangers with limited information about reputation. J. Public. Econ. 89, 1457–1468.

Brandt, H., Sigmund, K., 2004. The logic of reprobation: assessment and action rules for indirect reciprocity. J. Theor. Biol. 231, 475–486.

Brandt, H., Sigmund, K., 2005. Indirect reciprocity, image scoring, and moral hazard. Proc. Natl Acad. Sci. USA 102, 2666–2670.

Brandt, H., Sigmund, K., 2006. The good, the bad and the discriminator—errors in direct and indirect reciprocity. J. Theor. Biol. 239, 183–194.

Fehr, E., 2004. Don't lose your reputation. Nature 432, 449–450.

Fehr, E., Fischbacher, U., 2003. The nature of human altruism. Nature 425, 785–791.

Fishman, M.A., 2003. Indirect reciprocity among imperfect individuals. J. Theor. Biol. 225, 285–292.

Hofbauer, J., Sigmund, K., 1998. Evolutionary Games and Population Dynamics. Cambridge University Press, Cambridge.

Kandori, M., 1992. Social norms and community enforcement. Rev. Econ. Stud. 59, 63–80.

Leimar, O., Hammerstein, P., 2001. Evolution of cooperation through indirect reciprocation. Proc. R. Soc. London B 268, 745–753.

Lotem, A., Fishman, M.A., Stone, L., 1999. Evolution of cooperation between individuals. Nature 400, 226–227.

Milinski, M., Semmann, D., Bakker, T.C.M., Krambeck, H.J., 2001. Cooperation through indirect reciprocity: image scoring or standing strategy? Proc. R. Soc. London B 268, 2495–2501.

Milinski, M., Semmann, D., Krambeck, H.J., Marotzke, J., 2006. Stabilizing the Earth's climate is not a losing game: supporting evidence from public goods experiments. Proc. Natl Acad. Sci. USA 103, 3994–3998.

Nakamaru, M., Kawata, M., 2004. Evolution of rumors that discriminate lying defectors. Evol. Ecol. Res. 6, 261–283.

Nowak, M.A., Sigmund, K., 1998a. Evolution of indirect reciprocity by image scoring. Nature 393, 573–577.

Nowak, M.A., Sigmund, K., 1998b. The dynamics of indirect reciprocity. J. Theor. Biol. 194, 561–574.

Nowak, M.A., Sigmund, K., 2005. Evolution of indirect reciprocity. Nature 437, 1291–1298.

Ohtsuki, H., 2004. Reactive strategies in indirect reciprocity. J. Theor. Biol. 227, 299–314.

Ohtsuki, H., Iwasa, Y., 2004. How should we define goodness? Reputation dynamics in indirect reciprocity. J. Theor. Biol. 231, 107–120.

Ohtsuki, H., Iwasa, Y., 2006. The leading eight: social norms that can maintain cooperation by indirect reciprocity. J. Theor. Biol. 239, 435–444.

Panchanathan, K., Boyd, R., 2003. A tale of two defectors: the importance of standing for evolution of indirect reciprocity. J. Theor. Biol. 224, 115–126.

Panchanathan, K., Boyd, R., 2004. Indirect reciprocity can stabilize cooperation without the second-order free-rider problem. Nature 432, 449–450.

Sugden, R., 1986. The Economics of Rights, Cooperation and Welfare. Blackwell, Oxford.

Takagi, E., 1996. The generalized exchange perspective of the evolution of altruism. In: Liebrand, W.B.G., Messick, D.M. (Eds.), Frontiers in Social Dilemmas Research. Springer, Berlin, pp. 311–336.

Takahashi, N., Mashima, R., 2003. The emergence of indirect reciprocity: is the standing strategy the answer? Center for the study of cultural and ecological foundations of the mind, Hokkaido University, Japan, Working Paper Series No. 29.

Taylor, P.D., Jonker, L.B., 1978. Evolutionary stable strategies and game dynamics. Math. Biosci. 40, 145–156.

Wedekind, C., Braithwaite, V.A., 2002. The long-term benefits of human generosity in indirect reciprocity. Curr. Biol. 12, 1012–1015.

Wedekind, C., Milinski, M., 2000. Cooperation through image scoring in humans. Science 288, 850–852.